Vector Integration

Q. If $\vec{F} = (3x^2 + 6y)\vec{i} - 14yz\vec{j} + 20xz^2\vec{k}$ then evaluate $\int \vec{F} \cdot d\vec{r}$ along the straight line joining $(0,0,0)$ to $(1,1,1)$

Soln ⇒ Equation of st. line joining $(0,0,0)$ and $(1,1,1)$ is

$$\frac{x-0}{1} = \frac{y-0}{1} = \frac{z-0}{1}$$

$$\Rightarrow \frac{x}{1} = \frac{y}{1} = \frac{z}{1} = t \text{ (suppose)}$$

$\therefore x = t,\ y = t,\ z = t$

Also, $\vec{F} = (3t^2 + 6t)\vec{i} - 14t^2\vec{j} + 20t^3\vec{k}$

and $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k} = t(\vec{i} + \vec{j} + \vec{k})$

$\Rightarrow d\vec{r} = (\vec{i} + \vec{j} + \vec{k})\,dt$

$$\therefore \vec{F} \cdot d\vec{r} = (3t^2 + 6t)\,dt - 14t^2\,dt + 20t^3\,dt$$
$$= (20t^3 - 11t^2 + 6t)\,dt$$

$$\therefore \int_C \vec{F} \cdot d\vec{r} = \int_0^1 (20t^3 - 11t^2 + 6t)\,dt$$
$$= \left[5t^4 - \frac{11}{3}t^3 + 3t^2\right]_0^1$$
$$= 5 - \frac{11}{3} + 3 = 8 - \frac{11}{3} = \underline{\underline{\frac{13}{3}}}$$